Unit Circle Handout

The unit circle is commonly used to calculate the trigonometric functions Sin(x), Cos(x), and Tan(x) of an angle. The unit circle utilizes the (x, y) format to denote the points on the circle, where x represents Cos(θ) of a given angle, y represents Sin(θ), y/x represents Tan(θ), and θ is measured in degrees or radians. The degrees and radians are given at each individual coordinate. This handout will describe the unit circle concept, indicate how it is used, define degrees and radians, and explain how to convert degrees to radians.

What is the unit circle?
The unit circle is a circle with a radius of one. The center is known as the origin (0,0) which is the intersection of the x and y axes. The values listed on the unit circle are given in two separate units of measurement: degrees and radians. Degrees, denoted by °, are a measurement of angle size that is determined by dividing a circle into 360 equal pieces. Radians, which are unit-less but always written with respect to π, are a measurement of an angle in relation to a section of the unit circle’s circumference. An example expressing the difference between degrees and radians is shown below.

Circles can be divided into 360 degrees, which are measured by starting on the right side of the x-axis and moving counterclockwise until a full rotation has been completed. In radians, this would be equivalent to 2π. The circle to the left has been cut so that one fourth of it is missing, the section that has been removed is 90 degrees or \( \frac{\pi}{2} \).

In trigonometry, most calculations are computed in radians. Therefore, it is important to know how to convert from degrees to radians. The equation to convert from degrees to radians is as follows:

\[
\text{Radians} = \text{Degrees} \times \frac{\pi}{180}
\]
A full representation of the unit circle is given on the next page.

Key: (Cos(θ), Sin(θ))
Practice Problems:

Find the exact value of the problems below

1.) \( \sin \frac{4\pi}{3} \)
2.) \( \cos \frac{11\pi}{6} \)
3.) \( \tan \frac{\pi}{3} \)
4.) \( \cos \frac{-2\pi}{3} \)
5.) \( \sin \frac{-\pi}{2} \)
6.) \( \tan 2\pi \)
7.) \( \tan \frac{\pi}{2} \)

Answers:

1.) \( -\frac{\sqrt{3}}{2} \): \( \sin(\theta) \) is the \( y \) value of the coordinate
2.) \( \frac{\sqrt{3}}{2} \): \( \cos(\theta) \) is the \( x \) value of the coordinate
3.) \( \sqrt{3} \): \( \tan(\theta) \) is the \( y \) value divided by the \( x \) value
4.) \( -\frac{1}{2} \): Instead of rotating counter clockwise around the circle, go clockwise in the same amount of degrees or radians.
5.) \( -1 \): Same as before.
6.) \( 0 \): \( 0/1 \) is still \( 0 \)
8.) Undefined: Does not exist because \( \tan \frac{\pi}{2} \) equals \( \frac{1}{0} \) which cannot occur.