Graphing Linear Equations

Linear equations are used to form straight lines on a graph. The ability to graph a linear equation is essential to understanding and analyzing information. This handout will discuss the coordinate plane, how to plot points on the coordinate plane, and how to graph a linear equation in slope-intercept form.

The Coordinate Plane

The coordinate plane is a two-dimensional tool used to graph linear equations. It consists of a vertical line called the y-axis and a horizontal line called the x-axis. The point where the two lines intersect is called the origin, and all vertical and horizontal distances are plotted by counting units from the origin.

Once a coordinate system is established, points can be plotted that will provide the basis for graphing a line. A point is written in the format (x-value, y-value), which is known as an ordered pair. The x-value is the point’s distance from the origin in the x direction (horizontally), and the y-value is the point’s distance from the origin in the y direction (vertically).
**Example:** Plot the point (1, 2).
Step 1: Find the distance from the origin along the x axis.
Step 2: Find the distance from the origin along the y axis.
Step 3: Plot the point +1 horizontally and +2 vertically.

**Graphing Linear Equations Using Points**

In order to graph a linear equation, at least two points on the line must be found. By plugging in the x-value and solving for the y-value, a chart of ordered pairs can be created.

**Example:** Graph the linear equation \( y = x \)

Step 1: To find points on the line, begin by substituting a value for x to obtain a value for y; these two values create an ordered pair. It is typically easier to work with small integers. For instance, in this example, when the x-value equals -2 in the equation, the y-value equals -2. Therefore, the first ordered pair is (-2, -2). Substituting in the x-values -1, 0, 1, and 2 will result in the following table of ordered pairs:

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
Step 2: Plot the points of the table found in the previous step, and draw a line through them.

Notice that a line drawn through any two graphed points will also go through the rest of the graphed points. This means that only two points are needed to graph a line. If it is known that the points (0, 1) and (2, 2) are solutions to the linear equation \( y = \frac{1}{2} x + 1 \), then the rest of the equation’s solutions can be graphed by drawing a line that passes through these two points.

**Slope-Intercept Form**

Slope-intercept form is one way to graph linear equations and is represented with the formula \( y = mx + b \). In the equation, the “\( m \)” represents the slope, and the “\( b \)” represents the y-intercept.
Y-Intercept

The y-intercept is useful when graphing a linear equation because it provides a starting point to begin graphing. In order to plot the y-intercept point, it must be written as the ordered pair (0, b), where b is the value taken from the given slope-intercept equation.

Slope

The slope value of a linear equation indicates vertical and horizontal movement from a known point in order to plot a second point. The slope must be written as a fraction with the numerator corresponding to a vertical movement and the denominator corresponding to a horizontal movement on the coordinate plane. Instructors and textbooks often use the terminology “Rise over Run” as shown below:

Example 1: Graph the linear equation \( y = 2x + 3 \).

Step 1: Identify the ordered pair for the y-intercept. Since the given y-intercept value (b) in this equation is 3, the ordered pair for the y-intercept is (0, 3).

Step 2: Identify the slope value, and translate it into its fractional form. The slope value in this equation is the whole number 2, so it must be written as a fraction:

\[
\frac{2}{1}
\]

Following the terminology “Rise over Run,” the 2 is the “rise,” and the 1 is the “run.”
Step 3: Plot the y-intercept, and use the slope to plot a second point. After plotting the y-intercept (0, 3), rise 2 units in the positive y direction, and run one unit in the positive x direction to plot the next point (1, 5). Then, connect the points with a line.

In the example, the slope value is positive; therefore, the position of the second point is plotted vertically on the y-axis and to the right on the x-axis.

Example 2: Graph the linear equation:

\[ y = -\frac{2}{3}x + 1 \]

Step 1: Find the y-intercept, which is (0, 1).

Step 2: Identify the slope value, and translate it into its fractional form. The slope value in this equation is the fraction \(-\frac{2}{3}\). When dealing with a negative slope, the negative must be applied to the numerator of the fraction:

\[ -\frac{2}{3} \]
Step 3: Plot the y-intercept, then proceed 2 units down in the negative y direction, and 3 units right in the positive x direction. This gives the second point: (3,-1).
Practice Problems

Problem 1: Graph the Linear Equation $y = \frac{1}{2}x + 1$.

![Graph of $y = \frac{1}{2}x + 1$]

Problem 2: Graph the Linear Equation $y = -2x$.

![Graph of $y = -2x$]
Practice Problems Solutions

Problem 1: Graph the Linear Equation \( y = \frac{1}{2}x + 1 \).

Step 1: Obtain the y-intercept.
\[
y = \frac{1}{2}x + 1 \quad (0, 1)
\]
Step 2: Plot the y-intercept.
Step 3: Obtain the slope.
Slope = \( \frac{1}{2} \)
Step 4: Rise 1 and Run 2 from the y-intercept to plot the next point.
Step 5: Connect both points with a line.

Problem 2: Graph the Linear Equation \( y = -2x \).

Step 1: Obtain the y-intercept.
\[
y = -2x + 0 \quad (0, 0)
\]
Step 2: Plot the y-intercept.
Step 3: Obtain the slope.
Slope = -2
Step 4: Use the slope to determine the rise.
and run.
Rise = -2 and Run = 1
Step 5: Connect both points with a line.