

Polynomial Fractions

Simplifying

To simplify a polynomial fraction, start by factoring both the numerators and denominators completely. See our factoring handout if you need review on factoring methods. Next cancel out any common factors.

Example 1:

$$\frac{80x^2 + 80x}{30x + 30} \rightarrow \frac{80x(x+1)}{30(x+1)} \rightarrow \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot x \cdot (x+1)}{2 \cdot 3 \cdot 5 \cdot (x+1)}$$

$$\frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{5} \cdot x \cdot \cancel{(x+1)}}{\cancel{2} \cdot 3 \cdot \cancel{5} \cdot \cancel{(x+1)}} \rightarrow \frac{8x}{3}$$

Example 2: $\frac{6x^3 - 27x^2 + 30x}{2x^2 - 3x - 5} \rightarrow \frac{3x(2x^2 - 9x + 10)}{2x^2 - 3x - 5} \rightarrow \frac{3x(2x-5)(x-2)}{(2x-5)(x+1)}$

Factor

$$\rightarrow \frac{\cancel{3x}(\cancel{2x-5})(x-2)}{\cancel{(2x-5)}(x+1)} \rightarrow \frac{3x(x-2)}{x+1}$$

Cancel

Multiplying

To multiply polynomials, first factor both the numerator and denominator of both fractions. Second, combine the two fractions together. Third, cancel out like terms.

Example 3: $\left(\frac{x^2 + 8x + 15}{x + 6}\right)\left(\frac{x + 4}{x^2 + 7x + 12}\right) \rightarrow \left(\frac{(x+3)(x+5)}{x+6}\right)\left(\frac{x+4}{(x+4)(x+3)}\right)$

Factor Multiply

$$\rightarrow \frac{(x+3)(x+5)(x+4)}{(x+6)(x+4)(x+3)} \rightarrow \frac{\cancel{(x+3)}(\cancel{x+5})\cancel{(x+4)}}{(x+6)\cancel{(x+4)}\cancel{(x+3)}} \rightarrow \frac{x+5}{x+6}$$

Cancel

Example 4:

$$\left(\frac{3k}{x}\right)\left(\frac{x^2 + 5x}{3k^2}\right) \rightarrow \left(\frac{3k}{x}\right)\left(\frac{x(x+5)}{3k^2}\right) \rightarrow \frac{\cancel{3k}(\cancel{x})(x+5)}{\cancel{x}(\cancel{3k^2})} \rightarrow \frac{x+5}{k}$$

Factor Cancel

Dividing

When dividing polynomial fractions, first flip the second fraction and then multiply.
 See previous page for examples on multiplying.

Example 5:

$$\frac{2x(x+7)}{x^2-x} \div \frac{2x-6}{x-1} \rightarrow \left(\frac{2x(x+7)}{x^2-x}\right)\left(\frac{x-1}{2x-6}\right) \rightarrow \left(\frac{2x(x+7)}{x(x-1)}\right)\left(\frac{x-1}{2(x-3)}\right)$$

Flip the second term Factor Multiply

$$\rightarrow \frac{\cancel{2x}(x+7)\cancel{(x-1)}}{\cancel{2x}\cancel{(x-1)}(x-3)} \rightarrow \frac{x+7}{x-3}$$

Cancel

Adding/Subtracting

When adding and/or subtracting polynomial fractions, first get common denominators, then add and/or subtract the numerators together. Finally, simplify.

Example 6:

$$\frac{3}{5x} + \frac{4x}{3x^2} \rightarrow \left(\frac{3x}{3x}\right)\left(\frac{3}{5x}\right) + \left(\frac{4x}{3x^2}\right)\left(\frac{5}{5}\right) \rightarrow \frac{9x}{15x^2} + \frac{20x}{15x^2} \rightarrow \frac{29x}{15x^2} \rightarrow \frac{29}{15x}$$

Get common denominators Add Simplify

Example 7:

$$\frac{2x-2}{2x^2-x-1} - \frac{5x+1}{2x+1} \rightarrow \frac{2x-2}{(2x+1)(x-1)} - \frac{5x+1}{2x+1} \left(\frac{x-1}{x-1}\right) \rightarrow \frac{2x-2}{(2x+1)(x-1)} - \frac{5x^2-4x-1}{(2x+1)(x-1)}$$

Find common denominators Change subtraction to addition

$$\rightarrow \frac{2x-2}{(2x+1)(x-1)} + \frac{-(5x^2-4x-1)}{(2x+1)(x-1)} \rightarrow \frac{2x-2}{(2x+1)(x-1)} + \frac{-5x^2+4x+1}{(2x+1)(x-1)} \rightarrow \frac{-5x^2+6x-1}{(2x+1)(x-1)}$$

Distribute the negative Add numerators together Factor numerator

$$\rightarrow \frac{(-5x+1)(x-1)}{(2x+1)(x-1)} \rightarrow \frac{(-5x+1)\cancel{(x-1)}}{(2x+1)\cancel{(x-1)}} \rightarrow \frac{-5x+1}{2x+1}$$

Cancel

$(5x+1)(x-1)$	$-5x^2+6x-1$
$5x^2-5x+x-1$	$-5x^2+5x+1x-1$
$5x^2-4x-1$	$-5x(x-1)+1(x-1)$
	$(-5x+1)(x-1)$

Complex Fractions

To solve a complex fraction, first find the common denominator for the entire fraction. Then change each term's denominator to this common denominator. Once all the terms have the same denominator, the denominators can be removed.

Example 8:

$$\begin{array}{c}
 \frac{\frac{5x+6}{x+1} - 3}{6x + \frac{13x+15}{x+1}} \rightarrow \frac{\frac{5x+6}{x+1} - 3\left(\frac{x+1}{x+1}\right)}{\left(\frac{x+1}{x+1}\right)6x + \frac{13x+15}{x+1}} \rightarrow \frac{\frac{5x+6}{x+1} - \frac{3x+3}{x+1}}{\frac{6x^2+6x}{x+1} + \frac{13x+15}{x+1}} \\
 \text{Find Common Denominator} \qquad \qquad \qquad \text{Remove Denominators} \\
 \rightarrow \frac{5x+6-3x-3}{6x^2+6x+13x+15} \rightarrow \frac{2x+3}{6x^2+19x+15} \rightarrow \frac{2x+3}{(3x+5)(2x+3)} \rightarrow \frac{\cancel{2x+3}}{(3x+5)\cancel{(2x+3)}} \\
 \text{Simplify} \\
 \rightarrow \frac{1}{3x+5}
 \end{array}$$

Example 9:

$$\begin{array}{c}
 \frac{14 - \frac{43}{x} - \frac{21}{x^2}}{8 - \frac{22}{x} - \frac{21}{x^2}} \rightarrow \frac{x^2\left(14 - \frac{43}{x} - \frac{21}{x^2}\right)}{x^2\left(8 - \frac{22}{x} - \frac{21}{x^2}\right)} \rightarrow \frac{14x^2 - 43x - 21}{8x^2 - 22x - 21} \\
 \text{Multiply by the} \qquad \qquad \qquad \text{Factor} \\
 \text{common denominator} \\
 \rightarrow \frac{(2x-7)(7x+3)}{(2x-7)(4x+3)} \rightarrow \frac{\cancel{(2x-7)}(7x+3)}{\cancel{(2x-7)}(4x+3)} \rightarrow \frac{(7x+3)}{(4x+3)} \\
 \text{Cancel}
 \end{array}$$

Sample Problems

Write each fraction in lowest terms.

1. $\frac{90d^2}{80d^3}$

2. $\frac{25k^5}{30k^3}$

3. $\frac{-6k^6}{-11k}$

4. $\frac{72h^3}{81h^5}$

5. $\frac{54k^2}{18k}$

6. $\frac{-20i^2 + 200i}{-45i^2 + 475i - 250}$

7. $\frac{63h^3}{45h^2}$

8. $\frac{27g^4}{45g^5}$

9. $\frac{8b^2 + 6bw + w^2}{24b^2 + 54bw + 12w^2}$

Write each product in lowest terms.

10. $\frac{2k^2 + 3k}{6k^3 + 9k^2} * \frac{12v^2 - 28v}{4v}$

11. $\frac{15q^2 - 9q}{3q} * \frac{7g}{14g^3}$

12. $\frac{25g + 50}{g + 2} * \frac{g}{25(g - 2)}$

Complete and write in lowest terms.

13. $\frac{5k}{15} + \frac{4k}{5}$

14. $\frac{8b}{12} + \frac{9b}{6}$

15. $\frac{7f}{15} + \frac{5f}{5}$

16. $\frac{9e}{7} - \frac{8e}{21}$

17. $\frac{5j}{7} - \frac{7j}{21}$

18. $\frac{5k - 9}{4} - \frac{3k}{8}$

19. $\frac{7d}{7} - \frac{8d}{14}$

20. $\frac{5i}{16} - \frac{3i}{8}$

21. $\frac{3a + 4}{6} + \frac{4a - 14}{3}$

22. $\frac{6e}{5} + \frac{8e - 14}{20}$

23. $\frac{8}{r} + \frac{9}{f}$

24. $\frac{7j + 7}{6j} + \frac{6}{2u}$

25. $\frac{9a}{7} - \frac{6a}{28}$

26. $\frac{4}{d} - \frac{6}{y}$

27. $\frac{4t + 5}{2b} - \frac{-8t + 2}{16b}$

28. $\frac{6}{b} - \frac{3}{r}$

29. $\frac{5d}{7} - \frac{8d - 7}{28}$

30. $\frac{2}{v} + \frac{4}{a}$

Simplify

31.
$$\frac{4}{x^2 - 4} - \frac{3}{x^2 - x - 2}$$

32.
$$\frac{1}{a+b} - \frac{1}{a-b} + \frac{2a}{a^2 - b^2}$$

33.
$$\frac{2}{6-3x} + \frac{5}{x-2} - \frac{3}{4-2x}$$

34.
$$\frac{3}{2+a-6a^2} - \frac{1}{1+a-2a^2}$$

35.
$$\frac{x}{x^2+5x+6} + \frac{4}{x^2+6x+8} - \frac{2}{x^2+7x+12}$$

36.
$$\frac{72m^3n^4}{25p^2q^3} \div \frac{18m^2n^5}{35p^4q}$$

37.
$$\frac{(a-1)^2}{x} \div \frac{a^2-1}{ax}$$

38.
$$\frac{m^4-n^4}{m^4-m^2n^2} \div \frac{m^2+n^2}{m^3+m^2n}$$

39.
$$\frac{a^2+5a+6}{a^2-1} \cdot \frac{3+2a-a^2}{a^2-9}$$

40.
$$\frac{a^2-b^2}{a^2b^2} \cdot \frac{a^4}{a+b} \cdot \frac{b^2}{ab-a^2}$$

41.
$$\frac{a^2+ax}{3ax+2x^2} \cdot \frac{2ax-x^2}{ax+x^2} \div \frac{4a^2-2ax}{9a+6x}$$

42.
$$\frac{x^4-y^4}{4x^2-8xy+3y^2} \div \frac{x^2+y^2}{2x^2-3xy} \cdot \frac{2x-y}{x^2-xy}$$

43.
$$\frac{m^2+2m+1}{m^2+4m} \cdot \frac{m^2-16}{m^2-3m-4} \div \frac{m^2-2m-3}{m^2-m}$$

44.
$$\frac{a^2-ab-4a+4b}{4a^2-b^2} \cdot \frac{2a^2-ab}{a^2-ab-3a+3b}$$

45.
$$x^2 - \frac{x^3-1}{x + \frac{1}{x+1}}$$

46.
$$\frac{(a+b)^3}{a^3+b^3} \cdot \frac{a^2-ab+b^2}{a^2+2ab+b^2}$$

47.
$$\frac{a}{a-b} \cdot \frac{b-a}{a}$$

48.
$$\frac{a^3-3a^2-a+3}{14a^2b^2+14ab^2} \cdot \frac{21a^2b}{a^2-4a+3}$$

49.
$$\left(x - \frac{1}{x}\right) \div \left(x + \frac{1}{x} - 2\right)$$

50.
$$\left(8m - \frac{1}{m^2}\right) \div \left(4 - \frac{1}{m^2}\right)$$

51.
$$\frac{-5}{3-2x} + \frac{3}{2x-3} - \frac{x-3}{2x^2-x-3}$$

52.
$$\left(\frac{1}{x} - 1\right) \div (x-1)$$

Solutions

1. $\frac{9}{8d}$

2. $\frac{5k^2}{6}$

3. $\frac{6k^5}{11}$

4. $\frac{8}{9h^2}$

5. $3k$

6. $\frac{4i}{9i-5}$

7. $\frac{7h}{5}$

8. $\frac{3}{5g}$

9. $\frac{2b+w}{6(b+2w)}$

10. $\frac{3v-7k}{3k}$

11. $\frac{5q-3}{2g^2}$

12. $\frac{g}{g-2}$

13. $\frac{17k}{15}$

14. $\frac{13b}{6}$

15. $\frac{22f}{15}$

16. $\frac{19e}{21}$

17. $\frac{8j}{21}$

18. $\frac{7k-18}{8}$

19. $\frac{3d}{7}$

20. $\frac{11i}{16}$

21. $\frac{11a-24}{6}$

22. $\frac{16e-7}{10}$

23. $\frac{8f+9r}{rf}$

24. $\frac{7ju+7u+18j}{6ju}$

25. $\frac{15a}{14}$

26. $\frac{2(2y-3d)}{dy}$

27. $\frac{20t+19}{8b}$

28. $\frac{3(2r-b)}{br}$

29. $\frac{12d+7}{28}$

30. $\frac{2(a+2v)}{va}$

31. $\frac{1}{(x+2)(x+1)}$

32. $\frac{2}{a+b}$

33. $\frac{35}{6(x-2)}$

34. $\frac{1}{(1+2a)(1-a)(2-3a)}$

35. $\frac{1}{x+3}$

36. $\frac{28mp^2}{5nq^2}$

37. $\frac{a(a-1)}{a+1}$

38. $m+n$

39. $\frac{2+a}{1-a}$

40. $-a$

41. $\frac{3}{2x}$

42. $x+y$

43. $\frac{m-1}{m-3}$

44. $\frac{a(a-4)}{(a-3)(2a+b)}$

45. 1

46. 1

47. -1

48. $\frac{3a}{2b}$

49. $\frac{x+1}{x-1}$

50. $\frac{4m^2+2m+1}{2m+1}$

51. $\frac{7x+11}{(2x-3)(x+1)}$

52. $-\frac{1}{x}$