Statistics: Hypothesis Testing

Advertisers often make claims about products, such as the amount of bacteria an all-purpose cleaner can kill or the miles-per-gallon of a specific type of car. Did you know that these claims can be tested? In statistics, a **claim** is called a hypothesis. A **hypothesis** is an assumption about a population, whether that population consist of people, all-purpose kitchen cleaner, or cars. A **hypothesis test** tests the validity of the claim to see if there is enough evidence to support it.

This handout will define the basic elements of hypothesis testing, provide the steps to perform hypothesis tests using the **P-value** method and the **test statistic** method, as well as provide examples for conducting hypothesis tests with different methods. Many statistics courses use statistical calculation tools; however, this handout is designed for manually computed formulas.

**Basics of Hypothesis Testing**

All hypothesis tests consist of a **null hypothesis** (H₀) and an **alternate hypothesis**, which can be noted as H₁, Hₐ, or H₁. The null hypothesis is defined by an equality (=) in most cases, and the alternate hypothesis is written with an inequality (<, >, ≠). Consideration of a normal bell curve when conducting a hypothesis test is helpful.

The bell curve consists of three possible outcomes when conducting a hypothesis test: right-tailed test, left-tailed test, and the two-tailed test. The tail defines the area under the curve that fulfills the requirements of the claim, or the claim made by the hypothesis test. Below are diagrams of the three possible bell curves.
The decision of whether the claim requires a right-tailed, left-tailed, or two-tailed test is determined by the inequality sign in the alternate hypothesis as illustrated below. The direction of the tail helps determine **critical values**, which are used in both methods.

<table>
<thead>
<tr>
<th>Null hypothesis (H₀)</th>
<th>Alternative hypothesis (Hₐ, Hₐ, or H₁)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₀ will usually be <strong>equal</strong> to the claim, but can also be listed as ≥ or ≤.</td>
<td>Hₐ will use the symbols:</td>
</tr>
<tr>
<td></td>
<td>• &gt; for a right-tailed test</td>
</tr>
<tr>
<td></td>
<td>• &lt; for a left-tailed test</td>
</tr>
<tr>
<td></td>
<td>• ≠ for a two-tailed test</td>
</tr>
</tbody>
</table>

**The Difference between the Test Statistics and Z-Statistics**

Test statistics are used for different methods of hypothesis testing. There are three test statistic formulas which can be used to solve a hypothesis test; the test statistic formula that is used depends on the type of test being conducted. The z-stat formula requires the standard deviation (σ) be known, and the sample size (n) is greater than 30. The t-stat formula requires the σ to be unknown and n > 30.

Please refer to the following table for formulas when deciding which test statistic is required for a given problem. Assume the data is normal for these equations.

<table>
<thead>
<tr>
<th>Test Statistic Method</th>
<th>When to Use</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z statistic for mean</td>
<td>σ is known and n &gt; 30</td>
<td>$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$</td>
</tr>
<tr>
<td>Test statistic for mean</td>
<td>σ is unknown and n &gt; 30</td>
<td>$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$</td>
</tr>
<tr>
<td>Z statistic for proportion</td>
<td>A claim is made about a population proportion</td>
<td>$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$</td>
</tr>
</tbody>
</table>
Original Claim

Every hypothesis test has an original claim, which refers to the claim that someone is making and/or testing. This claim can usually be found in the last sentence of a hypothesis test.

**P-value Method**

In a hypothesis test problem, different pieces of information can be given in the problem to indicate that the *P-value* method should be used to test the validity of the claim of the hypothesis test. Sometimes a problem will include the *P-value*, while at other times, a problem will feature the data required to find a *P-value*. These may include: population standard deviation (σ), population mean (µ), and a population proportion (ρ) or a fraction \( \frac{3}{8} \).

**Example:**

The population proportion of dog owners in Virginia is 0.85. A 2500 person sample was gathered from Virginia, and the proportion of people who own dogs was found to be 0.84, with a sample mean of 0.52 and a sample standard deviation of 0.0145. Perform a hypothesis test at the 0.05 significance level (often denoted as \( \alpha \)) to test the claim that the population statistic is less than 0.85.

**Step 1: Determine Tail Direction and Original Claim**

The problem features the phrase “less than,” so the test is left-tailed. The original claim for this example problem is the alternate hypothesis because the final sentence states, “test the claim that the population statistic is less than 0.85.” The usage of “less than” shows that the original claim is the alternate hypothesis because the null hypothesis cannot contain a less than symbol.

**Step 2: Write the claims into the standard hypothesis test format:**

\[
H_0 : \ p = 0.85 \\
H_A : \ p < 0.85
\]
Step 3: Calculate the *P-value*. To calculate a *P-value*, use this systematic process:

A. **Choose a test statistic.** In this example, the test statistic needed is a z-score because the problem relates to proportions (refer to the test statistic table on page 2). The formula is: 

   \[ Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} \]

B. **Locate the required information to solve the formula.** In this example, the information needed is the sample proportion (\(\hat{p} = 0.84\)), population proportion (\(p = 0.85\)), and sample size \(n = 2500\).

C. **Solve the formula by inputting all required values to find the z-score.** In this problem, when rounded to two decimal places, the z-score is \(-1.40\).

D. **Compare the z-score to the z-score table in your textbook.** There are rules for finding a *P-value* based on if the test is right, left, or two-tailed. For a left-tail test, simply locate the *P-value* corresponding to the z-score. For a right-tail test, the *P-value* found in the table should be subtracted from 1. For a two-tailed test, multiply the *P-value* found in the table by 2.

E. **Compare the z-score to the *P-value*.** This example problem is a left-tailed test, and the *P-value*, when rounded to three decimal places, is 0.081.

Step 4: Decide whether to reject or not to reject the null hypothesis by comparing the *P-value* to the significance level (\(\alpha\)). Refer to the table below to make a decision about the comparison:

<table>
<thead>
<tr>
<th>(P\text{-value} \leq \alpha)</th>
<th>(P\text{-value} &gt; \alpha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject the null</td>
<td>Fail to reject the null</td>
</tr>
</tbody>
</table>

Because the *P-value* 0.081 is greater than \(\alpha\), which is 0.05, we must fail to reject the null hypothesis.
Step 5: Make a statement regarding the validity of the claim in the hypothesis test.

The statement will consist of two separate sentences: the first states the decision regarding the null hypothesis (Step 4), and the second states whether or not there is sufficient evidence to support the original claim of the problem. Below is an example of a statement for the sample problem.

There is not sufficient evidence to warrant rejection of the null hypothesis. Therefore, the sample data suggests there is sufficient evidence to support the claim that the proportion of people in Virginia who own dogs is 0.85.

Critical Value Method

This method is a way to test the validity of a claim by comparing a test statistic to a critical value.

Example:

People increasingly rely on their cell phones. A group of statisticians has sought to gather data about the cell phone usage in Virginia, and after exhaustive sampling, they have concluded that the mean amount of time people in Virginia spend on their cell phones per day is 180 minutes. Another group of statisticians gathered data from a 100-person sample group and concluded that the mean amount of time people use their cell phones in Virginia per day is 173 minutes. Assume the $\sigma$ is 50. Perform a hypothesis test at the 0.05 significance level to test the claim that the actual mean time that Virginians spend on their cell phones per day is less than 180 minutes.

Step 1: Determine Tail Direction and Original Claim

The problem includes the phrase “less than,” so the test is left-tailed. The original claim for this example problem is the alternate hypothesis because the problem states, “test the claim that the actual mean time that Virginians spend on their cell phones per day is less than 180 minutes.” The null hypothesis is always an equality, so the original claim, located in the final sentence of this problem, is the alternate hypothesis.
Step 2: Write out the Claim

\[ H_0 : \mu = 180 \]
\[ H_A : \mu < 180 \]

Step 3: Decide which Test Statistic to Use

Because \( \sigma \) is given, a z-stat is used (refer to the test statistic table on page 2 to help make a decision on which test statistic to use for a given problem).

Step 4: Input the values into the formula and solve

\[ Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{173 - 180}{50 / \sqrt{100}} \approx -1.4 \]

Step 5: Compare the Test Statistic to the Critical Value

Refer to the chart found in most statistics textbooks for critical z values in a z distribution. The critical value for a left-tailed test at a 0.05 level of significance is -1.645. Because the alternate hypothesis features a < symbol, any value below this number is in the rejection region and therefore, reject the null hypothesis. Note that -1.645 < -1.4. Because of this, the null hypothesis is rejected.

Step 6: Make a Statement about the Results

Based on the results of the previous steps, write a final statement for the claim. The following is an example statement for the sample problem:

There is not sufficient evidence to warrant rejection of the null hypothesis. Therefore, the sample data suggests there is insufficient evidence to support the original claim that the actual mean time of cell phone usage in Virginia is less than 180 minutes.